Contents

D_4

$$
\begin{aligned}
G &= \{a, b : a^4 = e, b^2 = e, bab^{-1} = a^{-1}\} \\
&= \{e, a, a^2, a^3, b, ab, a^2b, a^3b\}
\end{aligned}
$$

Let our set X be the verices of a square

$$X = \{1, 2, 3, 4\}$$

The orbit of each vertex x is the entire group since vertices can be sent to any other vertex through rotations,

$$O_x = \{1, 2, 3, 4\}$$

The stabilizer for 3 is

$$G_3 = \{e, b\}$$

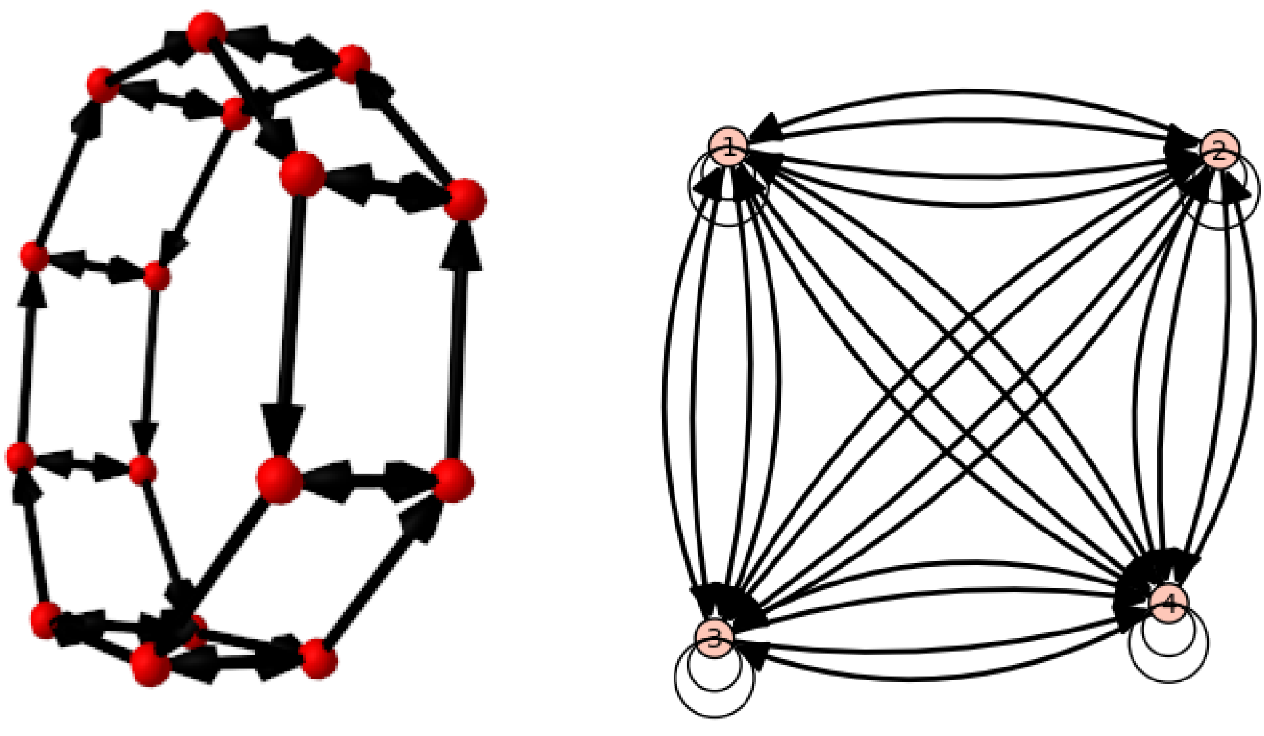
We now formalize this as per above.

$$X : \mathbf{B}G \to \mathbf{Set}$$

$$X(s) = \{1, 2, 3, 4\}$$

with X mapping morphisms identically. We now prove X is functorial. Let $g : s \to s$ define an arrow in $\mathbf{B}G$. Define the morphism $F(g) : \{1, 2, 3, 4\} \to \{1, 2, 3, 4\}$ by $F(g)(s) = m \cdot s$. Then

$$
\begin{aligned}
X(g_1 \circ g_2)(s) &= (m_1 m_2) \cdot s \\
&= m_1 \cdot (m_2 \cdot s) \\
&= X(m_1)(X(m_2)(s)) \\
&= (X(m_1) \circ X(m_2))(s)
\end{aligned}
$$



The entire group, and its skeletal translation groupoid.

The translation groupoid is the dihedral group applied to all sets S where $|S| = 4$. Which is an infinite collection of discrete identical graphs.

Each node has exactly 2 automorphisms:

```
1 ()
2 ()
3 ()
4 ()
1 (2,4)
3 (2,4)
2 (1,3)
4 (1,3)
```

Which are the stabilizers. All elements in the same orbit thus have isomorphic stabilizers.

The disjoint union of the stabilizer groups is:

$$\{(O_1, \{e, b\}), (O_3, \{e, b\}), (O_2, \{e, a^2b\}), (O_4, \{e, a^2b\})\}$$

The set of morphisms with domain x is isomorphic to G.

Another way of looking at it is the disjoint union of hom-sets $\mathrm{Hom}_{\mathbf{T}_G X}(x, y)$ for $y \in O_x$ where each set is isomorphic to $\mathrm{Hom}_{\mathbf{T}_G X}(x, x) = G_x$.

$$|G| = |O_x||G_x|$$